Funkcje zdaniowe

funkcje o wartościach logicznych (0/1, T/F)

$x^2 - 3 > 0$ ← To nie jest ZDANIE!

$\varphi(x) \equiv x^2 - 3 > 0$

$$\varphi(x) = \begin{cases} 1, & x > \sqrt{3} \vee x < -\sqrt{3} \\ 0, & x \in [-\sqrt{3}, \sqrt{3}] \end{cases}$$

Wykres f. zdaniowej

$S(\varphi) = \{x \in X : \varphi(x) = 1\}$

wykres

$S(\varphi) = (-\infty, -\sqrt{3}) \cup (\sqrt{3}, +\infty)$

$\varphi(x,y)$, $\varphi(x,y,t)$, $\varphi(x_1, \dots, x_n) \to \{0,1\}$

Kwantyfikatory

$\bigwedge$ ← „dla każdego"

ogólny

$\boxed{\bigwedge_x \varphi(x)}$ dla każdego $x \in X$ funkcja zdaniowa φ ma wartość 1

$\bigvee$ ← „istnieje"

szczegółowy

$\boxed{\bigvee_x \varphi(x)}$ istnieje $x \in X$, dla którego $\varphi(x) = 1$.

$\varphi(x) \equiv x^2 - 3 > 0, \quad \underline{x \in \mathbb{R} = X}$

$\underbrace{\bigwedge_x \underbrace{[x^2 - 3 > 0]}_{\text{f. zdaniowa}}}_{\text{zdanie}}$ – FAŁSZ

$\underbrace{\bigvee_x x^2 - 3 > 0}$ – PRAWDA

$\bigwedge_x \quad \forall_x \leftarrow$ for all

$\bigvee_x \quad \exists_x \leftarrow$ exists

$\bigwedge_x \varphi(x) \qquad \bigwedge_{x \in X} \varphi(x) \qquad \bigwedge_{x \in \boxed{\mathbb{Z}}} x^2 + 5 > 3$

$\bigwedge_x \bigvee_y \varphi(x, y) \leftarrow$ dla każdego x istnieje y, dla którego ...

Prawa rachunku kwantyfikatorów

$$\bigwedge_x \bigwedge_y \varphi(x,y) \equiv \bigwedge_y \bigwedge_x \varphi(x,y) \overset{\text{ozn}}{\equiv} \bigwedge_{x,y} \varphi(x,y)$$

$$\bigvee_x \bigvee_y \varphi(x,y) \equiv \bigvee_y \bigvee_x \varphi(x,y) \equiv \bigvee_{x,y} \varphi(x,y)$$

$$\bigwedge_x \bigvee_y \varphi(x,y) \Leftarrow \bigvee_y \bigwedge_x \varphi(x,y)$$

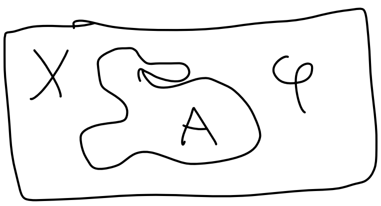
może zależeć od x

uniwersalny

$$\neg \bigwedge_x \varphi(x) \equiv \bigvee_x \neg \varphi(x)$$

$$\neg \bigvee_x \varphi(x) \equiv \bigwedge_x \neg \varphi(x)$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$



$$\bigwedge_x \left(x \in A \Rightarrow \varphi(x)\right) \overset{\text{ozn.}}{\equiv} \bigwedge_{x \in A} \varphi(x)$$

to nie jest uniwersum X

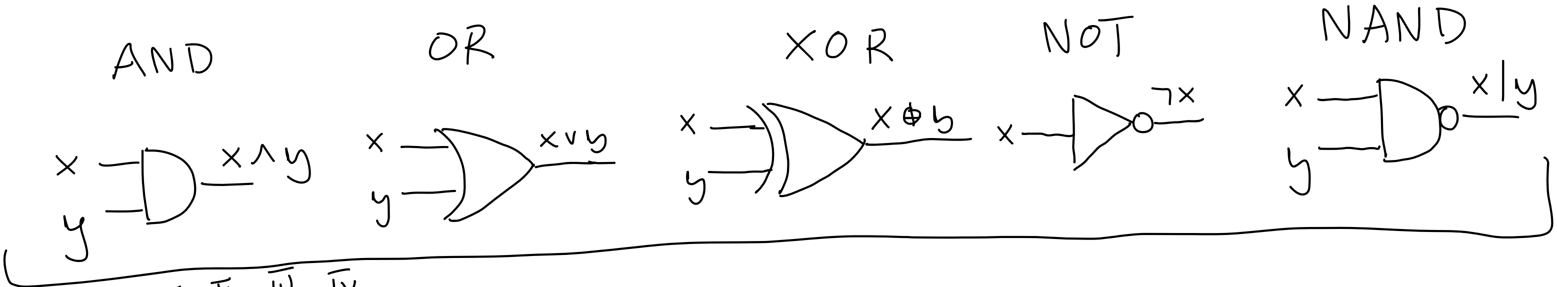
$$\bigwedge_{x \in (\sqrt{3}, +\infty)} x^2 - 3 > 0 .$$

```
   1 1 1 1
   1 0 1 1 1 0
+    1 0 1 1 1
--------------
 1 0 0 0 1 0 1
```

```
   1 1 1 1 0 0
   1 0 1 1 1 0
⊕    1 0 1 1 1
--------------
   1 1 1 0 0 1
⊕
 1 0 0 0 1 0 1
```

zupełny

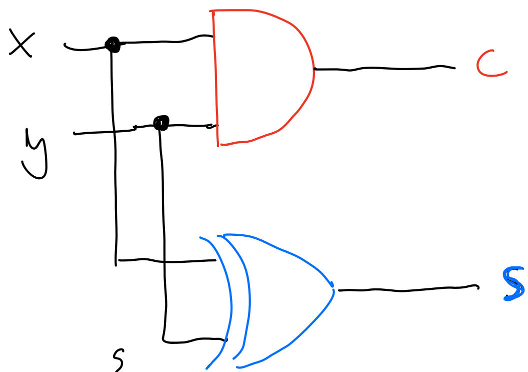
Bramki logiczne



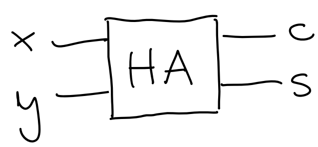
	I	II	III	IV
x	1	1	0	0
y	1	0	1	0
S	0	1	1	0
c	1	0	0	0

AND

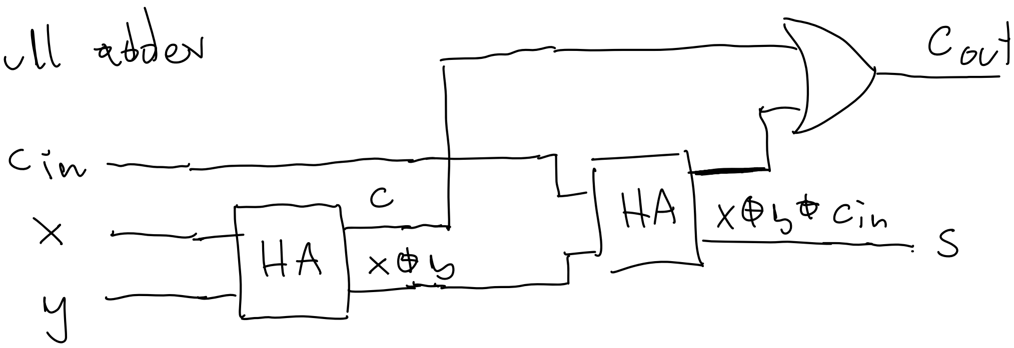
bit przeniesienia

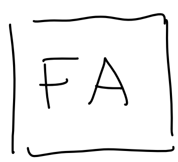


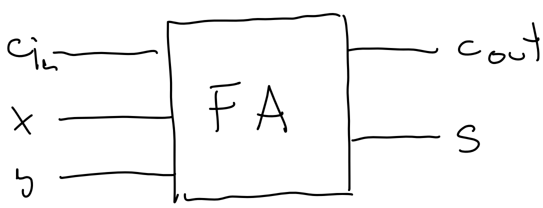
półsumator
half adder
HA



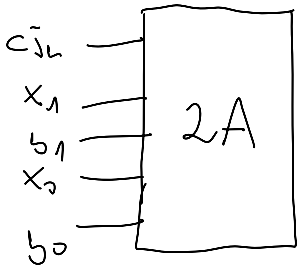
Full adder

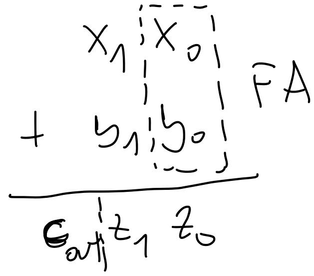


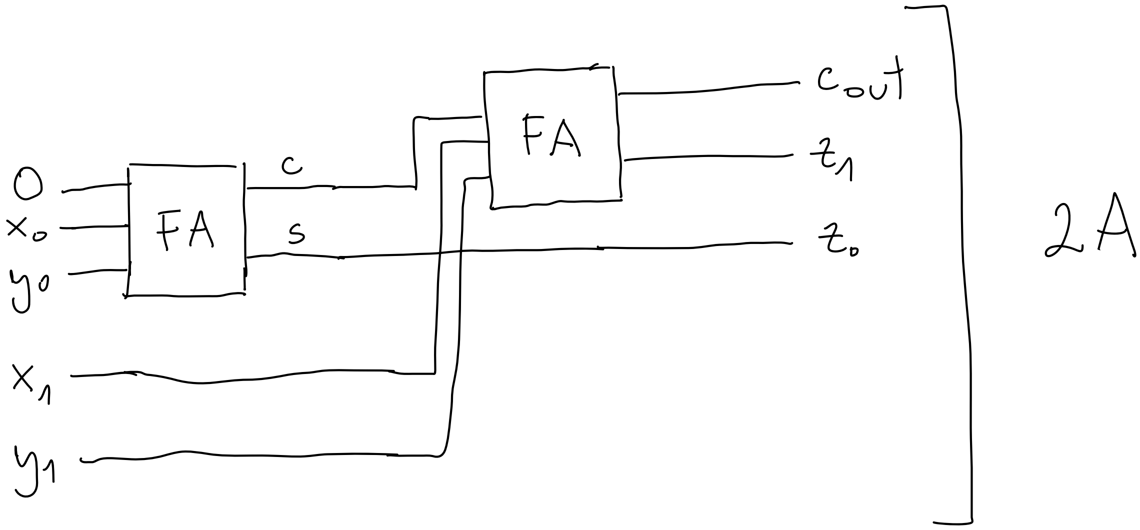


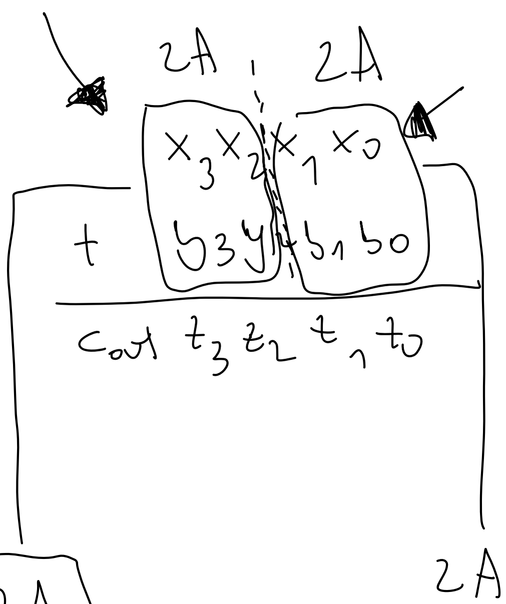


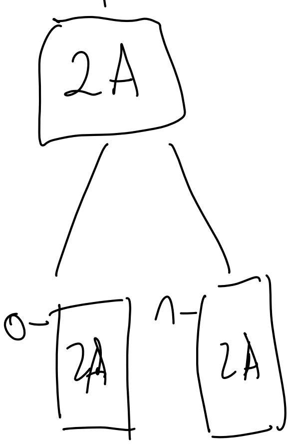
2A



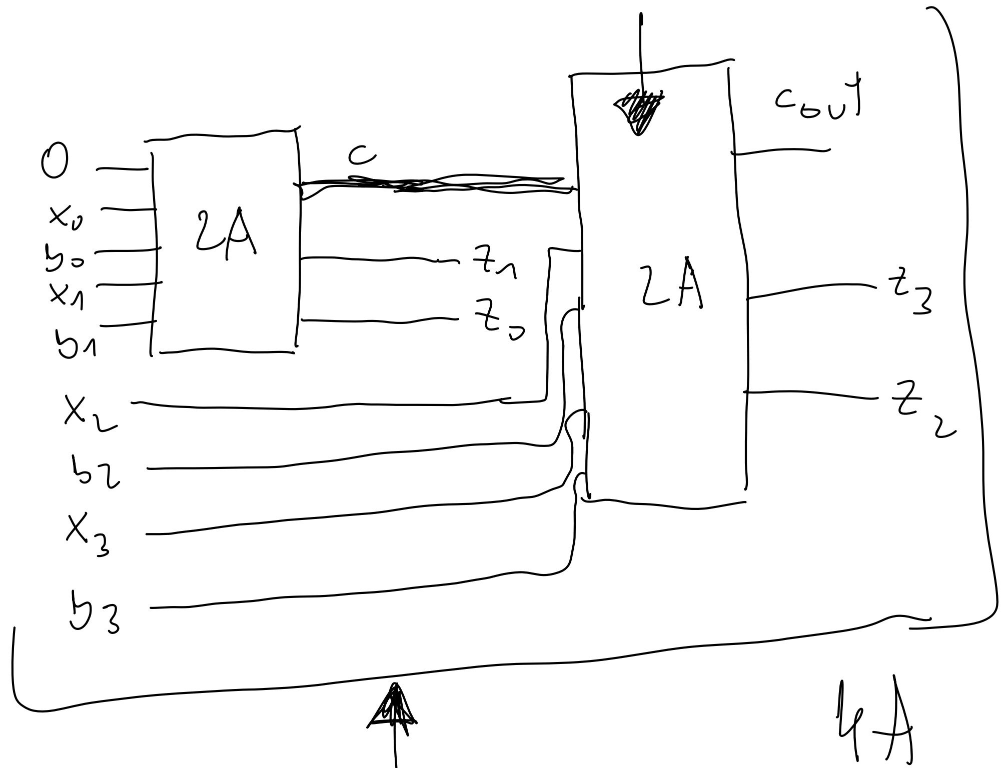


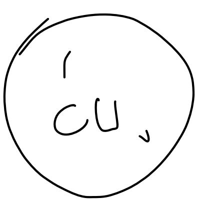


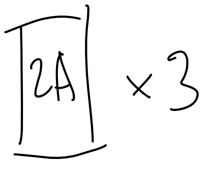




4A







optimizare

12

↓

8